

B.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2022.

Sixth Semester

Mathematics — Core

NUMERICAL METHODS

(For those who joined in July 2017 onwards)

Time : Three hours

Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

- The positive root of the equation $x^3 - x - 1 = 0$ lies between
(a) 1 and 2 (b) 0 and 1
(c) 2 and 3 (d) 3 and 4
- The order of convergence in Newton - Raphson method is _____
(a) 3 (b) 2
(c) 1 (d) 4

- When _____, Stirling's formula is used.

- (a) $-\frac{1}{2} < p < \frac{1}{2}$ (b) $p > \frac{1}{2}$
(c) $p > -\frac{1}{2}$ (d) $-1 < p < 1$

- The order of the error in Simpson's $\frac{1}{3}$ rule is _____

- (a) h (b) h^2
(c) h^3 (d) h^4

- If $f(0) = 1$, $f(1/3) = 0.75$, $f(2/3) = 0.6$, $f(1) = 0.5$, then the value of $\int_0^1 f(x) dx$ using Trapezoidal rule is

- (a) 0.7 (b) 0.6
(c) 0.8 (d) 1.6

- The particular integral of $y_{K+2} - 5y_{K-1} + 6y_K = 6^K$ is _____

- (a) 6^{K+1} (b) 6^{K-1}
(c) $\frac{6^{K-1}}{2}$ (d) $\frac{6^{K+1}}{2}$

- The value of $\Delta(3^x)$ is _____

- (a) 3^x (b) 3^{x+h}
(c) $3^x(3^h - 1)$ (d) $3^x - 1$

- $\Delta(\tan^{-1} x) =$ _____

- (a) $\tan^{-1}\left(\frac{h^2}{1+hx+x^2}\right)$
(b) $\tan^{-1}\left(\frac{h}{1-hx+x^2}\right)$
(c) $\tan^{-1}\left(\frac{h}{1+hx-x^2}\right)$
(d) $\tan^{-1}\left(\frac{h}{1+hx+x^2}\right)$

- From the following data

x : 5 15 22

y : 7 36 160

$\Delta y_7 =$ _____

- (a) 3.0 (b) 3.1
(c) 2.9 (d) 2.8

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- The order and degree of the equation $y_{x+2} - 3y_{x+1} + 5y_x = x^2$ are _____

- (a) 2, 3 (b) 3, 2
(c) 3, 3 (d) 2, 2

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

- (a) Find a real root of the equation $\cos x = 3x - 1$ correct to 3 decimal places by using iteration method.

Or

- (b) Determine the root of $xe^x - 3 = 0$ correct to three decimal places using the method of false position.

- (a) Evaluate $\Delta^{10}[(1-x)(1-2x^2)(1-3x^3)(1-4x^4)]$ if the interval of differencing is 2.

Or

- (b) Represent the function $x^4 - 12x^3 + 42x^2 - 30x + 9$ and its successive differences in factorial notation where the differencing interval $h = 1$.

13. (a) The following data gives the melting point of an alloy of lead and zinc, where t is the temperature in degree centigrade and p is the percentage of lead in the alloy.

p	40	50	60	70	80	90
t	184	204	226	250	276	304

Using Newton's interpolation formula, find the melting point of the alloy containing 84 percentage of lead.

Or

- (b) In the table below, estimate the missing value

x	0	1	2	3	4
y	1	2	4	-	16

Explain why it differs from $2^3 = 8$.

14. (a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1.25$ from the following data.

x	1.00	1.05	1.10	1.15
y	1.00000	1.02470	1.04881	1.07238
x	1.20	1.25	1.30	
y	1.09544	1.11803	1.4017	

Or

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- (b) Dividing the range into 10 equal parts, find the approximate value of $\int_0^{\pi} \sin x dx$ by Simpson's $\frac{1}{3}$ rd rule.

15. (a) Solve the difference equation $y_{n+1} - 2y_n \cos \alpha + y_{n-1} = 0$.

Or

- (b) Eliminate the constants from $y_n = A.2^n + B.3^n$ and derive the corresponding difference equation of the lowest possible order.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Find the inverse of the matrix $\begin{pmatrix} 4 & 1 & 2 \\ 2 & 3 & -1 \\ 1 & -2 & 2 \end{pmatrix}$ by using Gaussian elimination method.

Or

- (b) Find the negative root of the equation $x^3 - 2x + 5 = 0$.

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17. (a) Prove that

$$y_k = y_0 + k\Delta y_0 + \frac{k(k-1)}{1.2}\Delta^2 y_0 + \dots + \Delta^k y_0.$$

Or

- (b) Find the second difference of the polynomial $7x^4 + 12x^3 - 6x^2 + 5x - 3$ with interval of differencing $h = 2$.

18. (a) Prove that $y_p = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \dots +$

$$\frac{p(p+1)\dots(p+n-1)}{n!}\nabla^n y_n.$$

Or

- (b)

x	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	2028

From this table, find the value of $f(8)$ by using Newton's divided difference formula.

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19. (a) Find the Newton's backward difference formula to compute the derivatives.

Or

- (b) Using the following data, find $f'(5)$

x	0	2	3	4	7	9
$f(x)$	4	26	58	112	466	922

20. (a) Solve the equation

$$y_{n+2} + 2y_{n+1} - 56y_n = 2^n(n^2 - 3).$$

Or

- (b) Solve the difference equation $u(x+2) - 4u(x) = 9x^2$.

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